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Citation for published version:

Rice, WKM & Forgan, DH 2014, 'Convergence of Simulations of Self-Gravitating Accretion Discs - II: Sensitivity to the Implementation of Radiative Cooling and Artificial Viscosity', *Monthly Notices of the Royal Astronomical Society*, vol. 438, no. 2, pp. 1593-1602. <https://doi.org/10.1093/mnras/stt2297>

Digital Object Identifier (DOI):

[10.1093/mnras/stt2297](https://doi.org/10.1093/mnras/stt2297)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Publisher's PDF, also known as Version of record

Published In:

Monthly Notices of the Royal Astronomical Society

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Convergence of simulations of self-gravitating accretion discs – II. Sensitivity to the implementation of radiative cooling and artificial viscosity

W. K. M. Rice,^{1★} S.-J. Paardekooper,² D. H. Forgan¹ and P. J. Armitage^{3,4}

¹Scottish Universities Physics Alliance (SUPA), Institute for Astronomy, University of Edinburgh, Blackford Hill, Edinburgh EH9 3HJ, UK

²DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

³JILA, 440 UCB, University of Colorado, Boulder, CO 80309-0440, USA

⁴Department of Astrophysical and Planetary Sciences, University of Colorado, Boulder, CO 80309-0440, USA

Accepted 2013 November 27. Received 2013 November 22; in original form 2013 August 9

ABSTRACT

Recently, it has been suggested that the fragmentation boundary in smoothed particle hydrodynamic (SPH) and *FARGO* simulations of self-gravitating accretion discs with β -cooling do not converge as resolution is increased. Furthermore, this recent work suggests that by carefully optimizing the artificial viscosity parameters in these codes, it can be shown that fragmentation may occur for much longer cooling times than earlier work suggests. If correct, this result is intriguing as it suggests that gas giant planets could form, via direct gravitational collapse, reasonably close to their parent stars. This result is, however, slightly surprising and there have been a number of recent studies suggesting that the result is likely an indication of a numerical problem with the simulations. One suggestion, in particular, is that the SPH results are influenced by the manner in which the cooling is implemented. We extend this work here and show that if the cooling is implemented in a manner that removes a known numerical artefact in the shock regions, the fragmentation boundary converges to a value consistent with earlier work and that fragmentation is unlikely for the long cooling times suggested by this recent work. We also investigate the optimization of the artificial viscosity parameters and show that the values that appear optimal are likely introducing numerical problems in both the SPH and *FARGO* simulations. We therefore conclude that earlier predictions for the cooling times required for fragmentation are likely correct and that, as suggested by this earlier work, fragmentation cannot occur in the inner parts ($r < 50$ au) of typical protostellar discs.

Key words: accretion, accretion discs – gravitation – instabilities – planets and satellites: formation – stars: formation.

1 INTRODUCTION

If a disc around a central object is sufficiently massive, its own self-gravity may play an important role in its evolution through the growth of the gravitational instability (Safronov 1960; Goldreich & Lynden-Bell 1965). An infinitesimally thin disc is susceptible to the growth of an axisymmetric gravitational instability if the Q parameter (Toomre 1964)

$$Q = \frac{c_s \kappa}{\pi G \Sigma} < 1, \quad (1)$$

where c_s is the sound speed in the disc, κ is the local epicyclic frequency (equal to the angular velocity, Ω , in a Keplerian disc), G is the gravitational constant and Σ is the disc surface density. Global, non-axisymmetric perturbations can, however, grow for Q

values greater than 1, with simulations suggesting that the stability criterion in global discs is $Q < 1.5$ – 1.7 (Durisen et al. 2007).

It is now, however, quite well understood that the Q parameter alone does not determine the ultimate evolution of a self-gravitating accretion disc. The evolution is determined by both the value of Q and the rate at which the disc is able to lose energy (Pickett et al. 1998; Gammie 2001). The current picture is that for long cooling times, the disc will settle into a state of marginal stability (Paczynski 1978) in which the instability acts to transport angular momentum outwards, allowing mass to accrete on to the central object (Lin & Pringle 1987; Laughlin & Bodenheimer 1994; Lodato & Rice 2004; Mejía et al. 2005). For short cooling times, however, the disc may become sufficiently unstable to fragment and form bound objects (Kuiper 1951). This has been suggested as a mechanism for forming gas giant planets in discs around young stars (Boss 1998, 2000) or stars in discs around supermassive black holes (Shlosman & Begelman 1989; Goodman 2003; Bonnell & Rice 2008).

★E-mail: wkmr@roe.ac.uk

Two-dimensional, shearing sheet simulations (Gammie 2001), using a specific heat ratio of $\gamma = 2$, indicated that the boundary between fragmentation and a quasi-steady, self-gravitating state occurred at a cooling time of

$$\tau_c = 3\Omega^{-1}. \quad (2)$$

Rice et al. (2003) found a similar result using three-dimensional smoothed particle hydrodynamic (SPH) simulations. As already mentioned, in a quasi-steady state the gravitational instability acts to transport angular momentum outwards. In many instances, it is appropriate to assume that angular momentum transport is driven by disc viscosity which Shakura & Sunyaev (1973) suggest has the form

$$\nu = \alpha c_s H, \quad (3)$$

where $H = c_s/\Omega$ is the disc scaleheight. This form, however, assumes that the viscosity depends only on local disc properties. Given that disc self-gravity is inherently global, such a form is not necessarily suitable for characterizing angular transport in self-gravitating discs. However, if $Q \sim 1$ and if the disc mass is less than about half that of the central star, a local approximation appears to be a suitable representation (Balbus & Papaloizou 1999; Lodato & Rice 2004, 2005; Forgan et al. 2011a).

Since a quasi-steady state is one in which the cooling is balanced by an effective viscous heating, one can relate the viscosity to the cooling time through (Pringle 1981; Gammie 2001)

$$\alpha = \frac{4}{9\gamma(\gamma - 1)\tau_c\Omega}, \quad (4)$$

where γ is the specific heat ratio. Using different values of γ , Rice, Lodato & Armitage (2005) showed that rather than the fragmentation boundary depending on the cooling time, τ_c , it depends on the stresses in the disc, as represented by α . In agreement with Gammie (2001), their results indicate that self-gravitating discs can maintain a quasi-steady state if $\alpha < 0.06$ and will fragment if the required stress exceeds $\alpha > 0.06$.

Cossins, Lodato & Clarke (2009) used analytic calculations and three-dimensional numerical simulations to investigate further the energy balance in self-gravitating discs and found that the perturbation amplitude, $\delta\Sigma/\Sigma$, is related to the cooling time through

$$\frac{\delta\Sigma}{\Sigma} = \frac{1}{\sqrt{\beta_{\text{cool}}}}, \quad (5)$$

where $\beta_{\text{cool}} = \tau_c\Omega$. Using two-dimensional shearing sheet simulations, Rice et al. (2011) showed, similarly, that $\delta\Sigma/\Sigma \propto \alpha$. The basic picture that has therefore been developed is that a self-gravitating accretion disc will settle into a quasi-steady state in which cooling is balanced by heating driven by the gravitational instability. In such a state, the perturbation amplitudes will depend on the cooling rate (or, equivalently, on the level of stress in the disc) and if these perturbations are sufficiently large, they become non-linear, the disc is unable to maintain a quasi-steady state and instead fragments into bound objects. What makes this general picture attractive is that there is reasonable agreement across a wide range of different types of simulations including two-dimensional shearing sheet simulations (Gammie 2001; Rice et al. 2011), three-dimensional grid-based simulations (Mejía et al. 2005; Boley et al. 2007; Steiman-Cameron et al. 2013) and three-dimensional SPH simulations (Rice et al. 2005; Cossins et al. 2009).

Recent work (Meru & Bate 2011) has, however, shown that three-dimensional SPH simulations that fix β_{cool} do not converge to a well-defined fragmentation boundary as resolution is increased.

Their highest resolution simulations suggested that fragmentation could occur for cooling time $\tau_c > 10\Omega^{-1}$. Given that the Jeans mass of a typical fragment is well resolved in an SPH simulation even for quite modest resolutions (Bate & Burkert 1997; Rice, Forgan & Armitage 2012), this result is quite surprising. There have been a number of attempts to understand this result. Lodato & Clarke (2011) suggest that numerical viscosity may influence disc thermodynamics more than originally thought and hence that simulations may require higher resolutions than indicated by earlier calculations. Paardekooper, Baruteau & Meru (2011) use two-dimensional grid-based simulations to show that the lack of convergence could be related to edge effects in simulations with very smooth initial conditions. There is also some suggestion that disc fragmentation may have a stochastic nature (Paardekooper 2012).

Michael et al. (2012) considered convergence in three-dimensional, grid-based, self-gravitating disc simulations. This work, however, did not directly address convergence of the fragmentation boundary, but instead considered convergence of the properties of a quasi-steady, self-gravitating disc. Their results were consistent with fragmentation requiring $\alpha > 0.06$, but could not really make any strong statements about convergence of the fragmentation boundary. Steiman-Cameron et al. (2013) extended the work of Michael et al. (2012) to consider simulations with radiative cooling and found that the properties of these discs did not converge in the outer, optically thin regions. One of their conclusions was that there may be issues with convergence in regions where the optical depth is of order unity.

It has, however, been suggested (Rice et al. 2012) that the lack of convergence of the fragmentation boundary in the Meru & Bate (2011) simulations was simply a consequence of the manner in which the cooling was implemented. Rice et al. (2012) suggested that it might be related to the known problem – in many SPH implementations – of an unphysical discontinuity in the thermal energy (pressure) at contact discontinuities (Price 2008a,b) which, if not corrected for, could lead to regions with enhanced cooling. They also suggested that this could be solved by using a form of the cooling that smoothes across each SPH particle's neighbour sphere. In their simulations, fragmentation occurred only for cooling times $\tau_c < 9\Omega^{-1}$ but they could not claim convergence as their highest resolution simulations fragmented for a slightly longer cooling time than the value towards which the others appeared to be converging.

Meru & Bate (2012) have recently extended this convergence work to consider how it is affected by artificial viscosity in both SPH simulations and in FARGO grid-based simulations. They consider various artificial viscosity parameters and settle on the values that maximize the value of β_{cool} for which fragmentation can occur. In a quasi-steady state, the disc is in thermal equilibrium with the imposed cooling balanced by heating from both the instability and artificial viscosity. Ideally, the artificial heating should be minimized. The perturbation amplitudes should depend on the strength of the instability (Cossins et al. 2009; Rice et al. 2011), therefore choosing artificial viscosity parameters that maximize the value of β_{cool} at which fragmentation can occur should minimize the level of artificial heating.

There are, however, a few issues with this that will be discussed in more detail in a later section. Artificial viscosity is typically introduced so as to resolve shocks. In SPH, however, the artificial viscosity operates even in the absence of shocks, producing an artificial dissipation that should, ideally, be minimized. The way in which the instability heats the disc is through dissipation at shocks and so varying the viscosity parameters can reduce the artificial dissipation, but can also have an impact on the shock heating. Given

that Rice et al. (2012) suggest that the lack of convergence seen in Meru & Bate (2011) could be due to an unphysical structure in the shock regions, changing the shock structure could exacerbate this problem. Additionally, in FARGO, the artificial viscosity only operates at shocks and does not produce any kinematic viscosity, so should not introduce any artificial heating. Any artificial diffusion in FARGO should occur at the grid scale and therefore should not depend on the artificial viscosity parameters.

In this paper, we extend the work of Rice et al. (2012) to show that their suggested cooling formalism does indeed appear to lead to convergence of the fragmentation boundary and that, consistent with earlier work, fragmentation requires $\beta_{\text{cool}} < 8$ for $\gamma = 5/3$. We then extend this to consider how this result depends on the artificial viscosity in SPH to establish if the values suggested by Meru & Bate (2012) are indeed optimal. We also discuss their results obtained using FARGO. In Section 2, we briefly describe SPH. In Section 3, we consider the SPH results using the cooling formalism suggested by Rice et al. (2012). In Section 4, we address the results Meru & Bate (2012) obtained using FARGO and in Section 5 we discuss these results and conclude.

2 SMOOTHED PARTICLE HYDRODYNAMICS

2.1 The basic formalism

SPH is a Lagrangian hydrodynamic formalism in which a fluid, or gas, is represented by pseudo-particles (see, e.g. Hernquist & Katz 1989; Benz 1990; Monaghan 1992). Each particle is assigned a mass (m), position (x, y, z), velocity (v_x, v_y, v_z) and internal energy per unit mass (u). There are many descriptions of SPH (e.g. Benz 1990; Monaghan 1992), so we would not repeat the details here. Basically, the fluid/gas density is calculated via interpolation across the mass distribution. Pressure is determined via an equation of state. Gravitational forces can either be calculated by direct summation, or – more commonly – using a TREE code (Barnes & Hut 1986). The momentum and energy equations are in a form suitable for this Lagrangian formalism and so the particle velocities are updated using the gravitational and pressure forces on each particle, the positions are updated using the velocity of each particle, and the internal energy changes via PdV work, viscous dissipation and cooling.

2.2 Introducing cooling

To investigate the evolution of self-gravitating discs, a cooling time, τ_c , of the following form is typically used (Gammie 2001; Rice et al. 2003):

$$\tau_c = \beta_{\text{cool}} \Omega^{-1}. \quad (6)$$

This is typically added to the energy equation by assuming that the thermal energy of each particle, u_j , decays with an e-folding time given by τ_c . Hence, the energy equation becomes

$$\frac{du_j}{dt} = \frac{1}{2} \sum_i \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \mathbf{v}_{ij} \cdot \nabla_i W(|\mathbf{r}_i - \mathbf{r}_j|; h) - \frac{u_j}{\tau_c}, \quad (7)$$

where the sum is over all the neighbours, i , of particle j , \mathbf{v}_{ij} is the velocity difference between particle i and particle j , W is the smoothing kernel used to interpolate across the neighbours of particle j and h is the smoothing length that defines the volume of the neighbour sphere. As already mentioned, the unsmoothed – or

individual particle – internal energy has an unphysical discontinuity at the contact discontinuity behind shock waves (Price 2008a, 2008b). Consequently, Rice et al. (2012) suggested that the cooling be distributed across the neighbour sphere. Their suggestion was that the standard cooling term implementation

$$\left(\frac{du_j}{dt} \right)_{\text{cool}} = -\frac{u_j}{\tau_c} \quad (8)$$

should be replaced with

$$\begin{aligned} \left(\frac{du_j}{dt} \right)_{\text{cool}} &= -\frac{1}{\tau_c} \frac{W(0, h_j) m_j u_j}{\rho_j} \\ \left(\frac{du_{i \neq j}}{dt} \right)_{\text{cool}} &= -\frac{1}{\tau_c} \frac{m_i}{\rho_i} u_i W(|\mathbf{r}_j - \mathbf{r}_i|, h). \end{aligned} \quad (9)$$

Their suggestion was that the upper of the two equations in equation (9) would be applied to particle j , while the lower of the two equations would be applied to the neighbours i of particle j . This would ensure that the cooling rate associated with particle j would be

$$\left(\frac{du_j}{dt} \right)_{\text{cool}} = -\frac{1}{\tau_c} \sum_i \frac{m_i}{\rho_i} u_i W(|\mathbf{r}_j - \mathbf{r}_i|, h). \quad (10)$$

In regions without discontinuities, this will be the same as the cooling rate given by equation (8) (Rice et al. 2011). At the discontinuities in the shock regions, equation (10) will use the physically correct interpolated value for the internal energy and will ensure that the unphysical jump in thermal energy at the contact discontinuity does not artificially enhance the cooling in that region.

The results presented in Rice et al. (2012) using the cooling form shown in equation (9) suggested that the simulations were converging towards a fragmentation boundary between $\beta_{\text{cool}} = 6$ and 7. Their highest resolution simulation (10 million particles), however, fragmented between $\beta_{\text{cool}} = 8$ and 9 and hence they could not claim convergence. Their highest resolution simulation was, however, only a ring of 4 million particles that would have had the same resolution as a full 10 million particle simulation. Strictly speaking, it was not exactly the same conditions as the other lower resolution simulations. Here, we present results from a single 10 million particle run using the cooling form presented in equation (9) and that indicates a stronger constraint on the convergence of the fragmentation boundary.

2.3 Artificial viscosity

In SPH, the artificial viscosity has two main roles: to prevent particle interpenetration and to resolve shock waves. The viscosity can consequently be thought of as having a shear component and a bulk component (Monaghan 1985), with the bulk component acting very like a Von Neumann–Richtmeyer viscosity used to resolve shocks in many grid-based codes. A way in which to introduce viscosity in SPH, and what is used in all the simulations presented here, is to use the following to determine the viscosity term between particles i and j :

$$\Pi_{ij} = \begin{cases} \frac{-\alpha_{\text{SPH}} \bar{c}_{ij} \mu_{ij} + \beta_{\text{SPH}} \mu_{ij}^2}{\bar{\rho}_{ij}} & \mathbf{v}_{ij} \cdot \mathbf{r}_{ij} < 0; \\ 0 & \mathbf{v}_{ij} \cdot \mathbf{r}_{ij} > 0. \end{cases} \quad (11)$$

The terms \bar{c}_{ij} and $\bar{\rho}_{ij}$ are the average sound speed and density for particles i and j . The terms \mathbf{v}_{ij} and \mathbf{r}_{ij} are $(\mathbf{v}_i - \mathbf{v}_j)$ and $(\mathbf{r}_i - \mathbf{r}_j)$,

respectively. The coefficients α_{SPH} and β_{SPH} determine the strength of the two viscosity terms and the term μ_{ij} is given by

$$\mu_{ij} = \frac{h \mathbf{v}_{ij} \cdot \mathbf{r}_{ij}}{r_{ij}^2 + \eta^2}, \quad (12)$$

where η is a softening term that prevents the denominator in equation (12) from ever being zero. It should be clear that equation (11) only operates when particles are converging and hence will prevent interpenetration and resolve shocks. However, this form of the viscosity does not distinguish between converging flows and shear flows and hence this viscosity can also transport angular momentum and can, therefore, heat the system in the absence of shocks (Cartwright & Stamatellos 2010).

The viscosity coefficients typically satisfy $\beta_{\text{SPH}} = 2\alpha_{\text{SPH}}$ and commonly used values, in self-gravitating disc simulations, are $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.2$. One reason for using $\beta_{\text{SPH}} = 2\alpha_{\text{SPH}}$ is that it ensures that the first term in equation (11) dominates when the convergence is slow, while the second term dominates when convergence is rapid. The β_{SPH} term is essentially necessary so as to handle high Mach number shocks (Monaghan 1992). Consequently, most studies (Murray 1996; Lodato & Rice 2004) only consider the α_{SPH} term when determining the dissipation due to artificial viscosity in shear flows. Optimally, the α_{SPH} value should be set so as to minimize artificial dissipation while still preventing particle interpenetration.

Meru & Bate (2012) point out that the dissipation associated with the β_{SPH} term is not actually negligible but is about a factor of 2 smaller than that associated with the α_{SPH} term when $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.2$. They, therefore, conclude that one should optimize in terms of both α_{SPH} and β_{SPH} and conclude that the optimal values are $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 2$. This was largely based on simulations that maximized the value of β_{cool} – which determines the cooling time – for which fragmentation could occur. Maximizing the cooling time at which fragmentation occurs suggests that one has minimized the amount of artificial dissipation and so is an attractive strategy to adopt. However, as we will discuss in more detail later, this may not necessarily be the case and so, here, we investigate how these values of α_{SPH} and β_{SPH} influence the fragmentation boundary when using the modified cooling form proposed by Rice et al. (2012).

2.4 Simulation setup

All of the SPH simulations presented here have the same basic setup as those presented by Meru & Bate (2012). They have a central star with mass $M_* = 1$ surrounded by a disc extending from $r_{\text{in}} = 0.25$ to $r_{\text{out}} = 25$, with a mass of $M_{\text{disc}} = 0.1M_*$, an initial surface density profile of $\Sigma \propto r^{-1}$ and an initial minimum Q parameter of $Q = 2$. We impose a cooling of the form described by equation (6), but that is either implemented as in Meru & Bate (2012) – which we call basic cooling – or in the modified manner suggested by Rice et al. (2012) – which we call smoothed cooling. We consider various resolutions, ranging from 250 000 particles to 10 million particles. In all our simulations, we take $\alpha_{\text{SPH}} = 0.1$, but consider both $\beta_{\text{SPH}} = 0.2$ and 2.

3 RESULTS

3.1 Convergence using smoothed cooling

In their work, Rice et al. (2012) considered full simulations using 250 000, 500 000 and 2 million particles, but represented a 10 million particle simulation using a simulation with 4 million particles

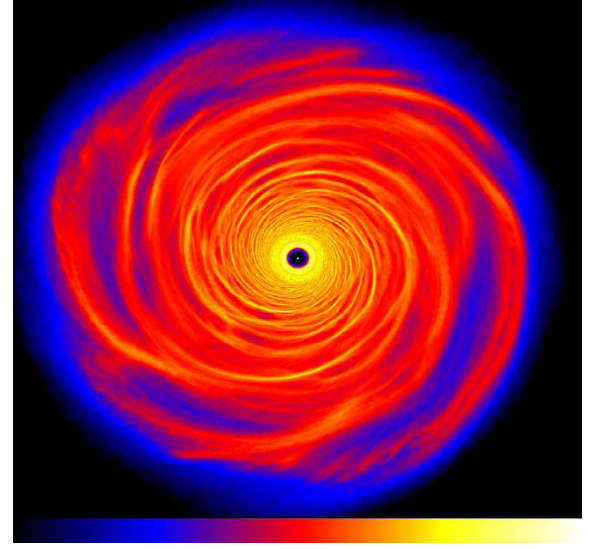


Figure 1. Final state of a full 10 million particle simulation using smoothed cooling with $\beta_{\text{cool}} = 8$. At this stage (after 6.5 outer rotation periods), the disc has settled into a quasi-steady state and there is no evidence of fragmentation.

Table 1. List of the simulations using smoothed cooling.

Simulation	No. of particles	β_{cool}	Fragment?
1	250 000	4	Yes
2	250 000	4.5	Yes
3	250 000	5	No
4	250 000	6	No
5	250 000	7	No
6	500 000	5	Yes
7	500 000	6	Yes
8	500 000	7	No
9	500 000	8	No
10	2000 000	5	Yes
11	2000 000	6	Yes
12	2000 000	7	No
13	2000 000	8	No
14	10 000 000	8	No

with a mass of $M_{\text{disc}} = 0.04M_*$ and that extended from $r_{\text{in}} = 15$ to $r_{\text{out}} = 25$. These parameters were chosen so as to have the same properties, in that region, as a full 10 million particle simulation. In Rice et al. (2012), the 500 000 and 2 million particle simulations fragmented at between $\beta_{\text{cool}} = 6$ and 7 and appeared to be converging. The pseudo-10 million particle simulation, however, fragmented between $\beta_{\text{cool}} = 8$ and 9 and hence they could not claim convergence.

We have since managed to complete a full 10 million particle simulation with $\beta_{\text{cool}} = 8$ which, after 6.5 outer rotation periods, shows no signs of fragmentation. The state of the simulation at this time (5120 code units or 815 orbits at $r = 1$) is shown in Fig. 1. There is clearly lots of spiral structure, but no evidence of fragmentation. Table 1 includes the results from the simulations of Rice et al. (2012) together with this new result using 10 million particles.

Fig. 2 is an updated version of that presented by Rice et al. (2012). It shows β_{cool} plotted against the particle number. The squares are for those simulations with the largest values of β_{cool} that fragmented and in which the fragments survived and became sufficiently dense that the simulation effectively stopped. The triangles

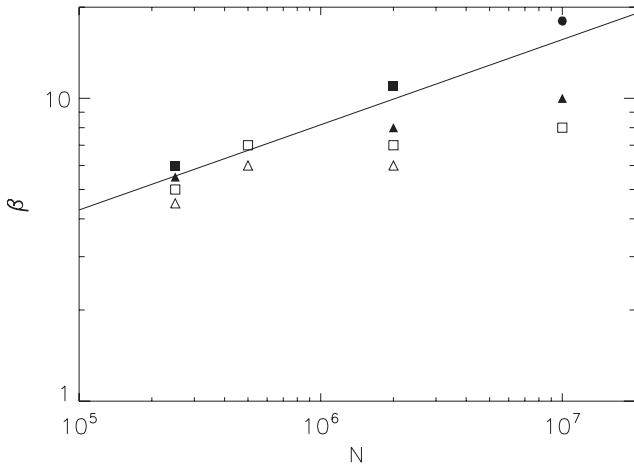


Figure 2. Fragmentation boundary from Meru & Bate (2011) (filled symbols) together with the results from Rice et al. (2012) and the new 10 million particle result from this work (open symbols). The triangles are for the largest value of β_{cool} for which fragmentation occurred. The squares are for the smallest value of β_{cool} for which a quasi-steady state is reached. The circle is for one simulation from Meru & Bate (2011) which was regarded as borderline. The line is from Meru & Bate (2011) and illustrates the lack of convergence seen in their simulations. The results from Rice et al. (2012) together with the new result from this work would indicate that the fragmentation boundary has converged to a value of $\beta_{\text{crit}} < 8$.

are for those with the lowest values of β_{cool} that did not fragment. The single filled circle is for a simulation that was regarded as borderline (Meru & Bate 2011). The data points are from Rice et al. (2012) (open symbols and particle numbers of 250 000, 500 000 and 2 million), Meru & Bate (2011) (filled symbols) and this work (10 million particle simulation).

The line in Fig. 2 is the same as that included by Meru & Bate (2011) to show that their simulations do not converge to a well-defined value of β_{crit} that defines the fragmentation boundary. In their later work (Meru & Bate 2012), however, they carry out many more simulations and suggest that the fragmentation boundary is actually converging towards $\beta_{\text{crit}} \sim 17.4$. However, even their highest resolution simulations (16 million particles) have not technically converged as they fragment between $\beta_{\text{cool}} = 10$ and 12. Fig. 2 shows that using smoothed cooling (which should only differ from basic cooling in the shock regions) results in fragmentation requiring $\beta_{\text{cool}} < 8$ for all resolutions considered. To have a better sense of whether these have converged or not, we should probably do a 10 million particle simulation with $\beta_{\text{cool}} = 7$, but this would take a significant amount of time and it already seems clear that smoothed cooling results in fragmentation requiring $\beta_{\text{cool}} < 8$ for all particle numbers considered and that the converged value would be between $\beta_{\text{cool}} = 6$ and 8.

Fig. 2 also suggests that simulations with 500 000 particles are close to being converged, if not actually converged. This is numerically sensible given that with $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.2$, artificial dissipation should be providing less than 10 per cent of the heating for $\beta_{\text{cool}} < 10$ (Lodato & Rice 2004). Furthermore, the outer half of such a disc resolves the Jeans mass at $Q = 1$ (Rice et al. 2012) and so should suitably resolve any possible fragmentation. Given that the smoothed cooling formalism seems like a reasonable manner in which to impose the cooling and which should only differ from basic cooling in the shock regions (where it will act to remove the unphysical discontinuity in thermal energy at the contact discontinuity), one could conclude that fragmentation requires (for

$\gamma = 5/3$) $\beta_{\text{cool}} < 8$ and that the lack of convergence seen in Meru & Bate (2011, 2012) is an entirely numerical artefact related to the manner in which they impose their cooling.

We should add that if our interpretation of the reason for this convergence issue has merit, it may be possible to continue using the basic cooling formalism as long as some form of heat conduction is included in the SPH simulations so as to remove the non-physical jump in the thermal energy at contact discontinuities (Price 2008a). We plan to investigate this possibility in future work.

3.2 The influence of artificial viscosity – basic cooling

To investigate the lack of convergence of the fragmentation boundary in SPH simulations with β -cooling, Meru & Bate (2012) considered the influence of artificial viscosity. As discussed earlier, the artificial viscosity has two terms, one that largely acts to prevent particle interpenetration and one that acts to resolve high Mach number shocks. In principle, the artificial viscosity should only act on converging flows. However, in a disc simulation it is unable to distinguish between converging flows and shear flows and so it also produces a shear viscosity that transports angular momentum and hence, even in the absence of shocks, produces dissipation. Ideally, the artificial viscosity parameters should be optimized so as to minimize the level of artificial dissipation and hence ensure that ‘real’ dissipation (such as that associated with the gravitational instability) dominates.

Previous work (Lodato & Rice 2004; Rice et al. 2005; Cossins et al. 2009) has used $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.2$. Meru & Bate (2012) vary α_{SPH} and β_{SPH} , in simulations with 250 000 particles, to consider how these parameters influence the fragmentation boundary. The goal, in some sense, is to maximize β_{crit} (i.e. the critical value of β_{cool} at which fragmentation occurs) as this would imply that this has minimized the level of artificial dissipation. With fixed β_{SPH} they find that β_{crit} is maximized for $\alpha_{\text{SPH}} = 0.1$. For fixed α_{SPH} , they claim that their results suggest that the optimal value for β_{SPH} is $\beta_{\text{SPH}} = 2$. However, their own figure suggests that β_{crit} is approximately constant for $\beta_{\text{SPH}} < 0.2$, that it rises as β_{SPH} is increased from 0.2 to 2 and then become constant again for $\beta_{\text{SPH}} > 2$. They have not really maximized β_{crit} ; they appear to have found a plateau. This would suggest that any value of $\beta_{\text{SPH}} > 2$ would be suitable, which is a little surprising as β_{SPH} essentially resolves shocks and determines the shock structure. Larger values will tend to result in broader shock fronts and so the norm would be to make β_{SPH} as small as possible. That their results suggest a step change between $\beta_{\text{SPH}} = 0.2$ and $\beta_{\text{SPH}} = 2$ might indicate a numerical issue, rather than an optimization of β_{SPH} .

Meru & Bate (2012) then repeated their earlier simulations (Meru & Bate 2011) using $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 2$. These simulations resulted in values for β_{crit} that were typically at least 50 per cent greater than that obtained using $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.2$. Their analysis suggested that these simulations were converging towards $\beta_{\text{crit}} = 29.2$, considerably higher than the value ($\beta_{\text{crit}} = 17.4$) obtained (Meru & Bate 2011) with basic cooling using $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 0.2$. This would tend to imply that the larger value of β_{SPH} has resulted in a reduced level of artificial dissipation.

There are, however, a number of issues with this interpretation. For example, one would expect the level of artificial dissipation to decrease as the particle number increases. The value of β_{crit} should therefore converge, for large enough particle number, to a value independent of β_{SPH} . The Meru & Bate (2012) analysis does not seem to show any such convergence. Additionally, if the correct

value of β_{crit} is more than 50 per cent higher than that suggested by earlier SPH simulations, this would imply that artificial dissipation provided at least one-third of the heating in these earlier SPH simulations. Earlier work (Lodato & Rice 2004) considered the level of artificial dissipation and estimated, for simulations with 250 000 particles, that it would provide less than 10 per cent of the dissipation for $\beta < 10$.

Forgan et al. (2011a) used 500 000 particle SPH simulations to investigate how self-gravitating discs evolve in the presence of realistic cooling. Their estimate of the level of artificial dissipation using the α_{SPH} term only, as suggested by Murray (1996), leads them to suggest that the artificial dissipation would dominate inside 10–20 au [consistent with a similar analysis by Clarke (2009)]. This was consistent with the results of their simulations, as the gravitational stresses very quickly became negligible inside 10–20 au (Forgan et al. 2011a). The simulations by Forgan et al. (2011a) are not, however, consistent with artificial dissipation being at least a factor of 3 higher than that expected as that would have led to the artificial dissipation dominating inside 30 au, rather than only inside 10–20 au.

It has, however, been shown that there are circumstances in which β_{SPH} values of 2 or greater may be necessary so as to reduce the level of unphysical, random particle motions (Price & Federrath 2010). However, this appears to be mainly relevant for high Mach number flows ($M \sim 10$). The turbulent velocities in self-gravitating disc simulations are typically subsonic (Forgan, Armitage & Simon 2011b) and so it is likely that the linear SPH artificial viscosity term (α_{SPH}) will play the dominant role in reducing this particle noise. Furthermore, the only source of energy for random particle motions is the kinetic energy of the flow itself. Therefore, if such artificial turbulence was being generated and dissipated, this should transport angular momentum and be reflected in the calculated stresses. As discussed above, however, previous work (Lodato & Rice 2004; Forgan et al. 2011a) appears not to be consistent with the level of such turbulence being significantly greater than basic estimates suggest (Murray 1996).

3.3 The influence of artificial viscosity – smoothed cooling

By varying the artificial viscosity parameters, Meru & Bate (2012) have suggested that the fragmentation boundary, as parametrized by β_{crit} , is at least 50 per cent higher than previously thought. As suggested above, however, this does appear to be inconsistent with earlier work (Lodato & Rice 2004; Forgan et al. 2011a). Their choice of artificial viscosity parameters was, however, motivated by a sense that these parameters minimized the level of artificial dissipation. Consequently, if this is indeed the case, we would expect to see a similar result if we used these parameters together with the smoothed cooling suggested by Rice et al. (2012) (see equation 9).

To investigate the influence of artificial viscosity when using smoothed cooling, we repeated the 250 000, 500 000 and 2 million particle simulations, but with $\alpha_{\text{SPH}} = 0.1$ and $\beta_{\text{SPH}} = 2$. Fig. 3 shows the final state for two 500 000 particle simulations, both of which used $\beta = 8$. In the top panel, $\beta_{\text{SPH}} = 0.2$, while in the bottom panel $\beta_{\text{SPH}} = 2$. Although they are similar, there are clear differences. Using $\beta_{\text{SPH}} = 2$ has removed some of the noise present in the outer parts of the $\beta_{\text{SPH}} = 0.2$ simulation. However, although there is a coherent spiral structure in the inner parts of the $\beta_{\text{SPH}} = 0.2$ simulation, it is not present in the $\beta_{\text{SPH}} = 2$ simulation. Additionally, the inner hole is larger when $\beta_{\text{SPH}} = 2$ than when $\beta_{\text{SPH}} = 0.2$. This is consistent with shocks being more smeared out when a larger β_{SPH} value is used and is consistent with the larger β_{SPH} producing

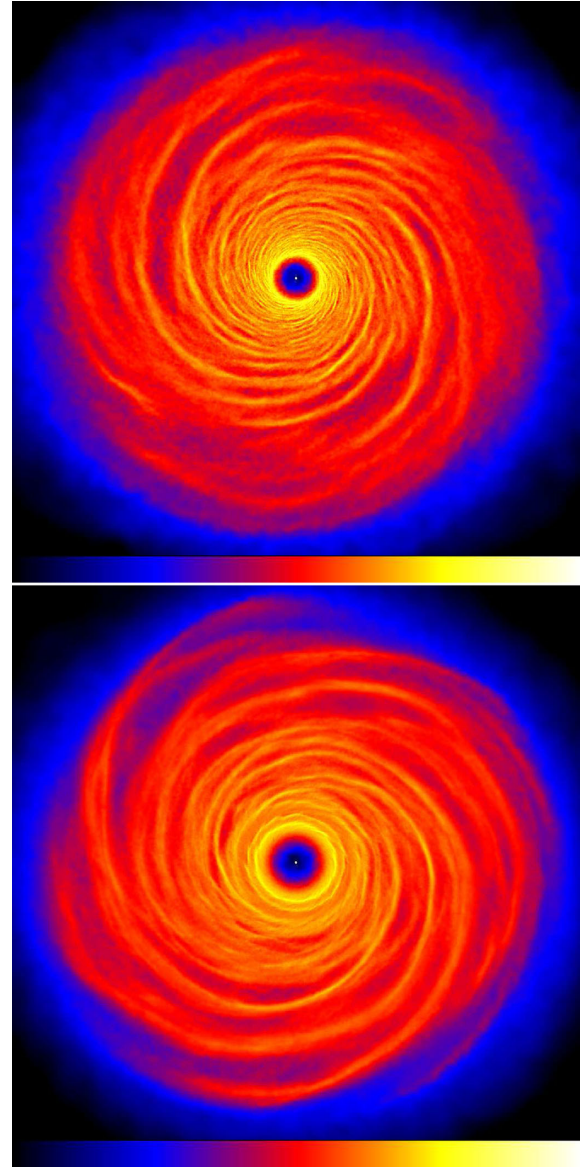


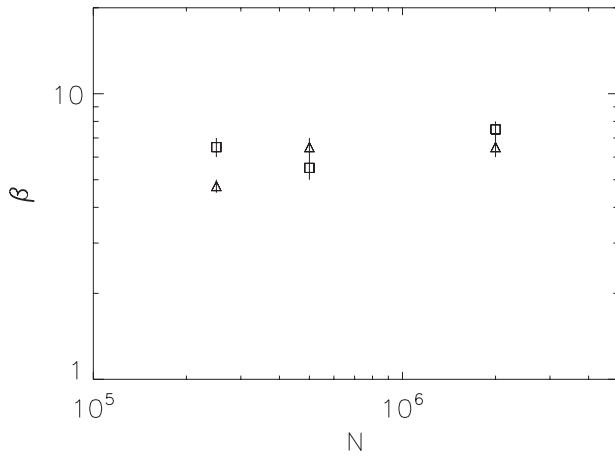
Figure 3. The final states of two 500 000 particles simulations both of which used $\beta_{\text{cool}} = 8$ with smoothed cooling. The top figure is from a simulation using $\beta_{\text{SPH}} = 0.2$, while the bottom is from one that used $\beta_{\text{SPH}} = 2$. Although similar, there are clear differences. Using $\beta_{\text{SPH}} = 2$ seems to reduce the level of noise in the outer parts of the disc, but also smears out the spiral structure in the inner regions and increases the size of the inner hole.

a larger artificial viscosity (and hence clearing out more of the inner disc).

Table 2 shows the results of the smoothed cooling simulations using $\beta_{\text{SPH}} = 2$. Fig. 4 compares the results using $\beta_{\text{SPH}} = 0.2$ (triangles) with those obtained using $\beta_{\text{SPH}} = 2$ (squares). The symbols are located at the average of the maximum β_{cool} for which fragmentation occurred and the minimum for which it did not. The short lines indicate the range between these two values. Although there is a difference, the results are very similar. There is no indication that β_{crit} increases by 50 per cent when $\beta_{\text{SPH}} = 2$, compared to that obtained when $\beta_{\text{SPH}} = 0.2$. Given that previous analysis (Meru & Bate 2012) indicates that $\beta_{\text{SPH}} = 2$ should produce more artificial dissipation than $\beta_{\text{SPH}} = 0.2$, it is little surprising that β_{crit} is not smaller when using $\beta_{\text{SPH}} = 2$ than when using $\beta_{\text{SPH}} = 0.2$. It is

Table 2. List of the simulations using smoothed cooling with $\beta_{\text{SPH}} = 2$.

Simulation	No. of particles	β_{cool}	Fragment?
1	250 000	4	Yes
2	250 000	5	Yes
3	250 000	6	No
4	250 000	7	No
6	500 000	5	Yes
7	500 000	6	No
8	500 000	7	No
11	2000 000	7	Yes
12	2000 000	8	No

**Figure 4.** Comparison of the fragmentation boundary obtained using smoothed cooling with $\beta_{\text{SPH}} = 2$ (squares) and with $\beta_{\text{SPH}} = 0.2$ (triangles). The symbols are located at the average of the maximum β_{cool} for which fragmentation occurred and the minimum for which it did not, while the lines indicate the range between these two values.

possible that using $\beta_{\text{SPH}} = 2$ does reduce some of the random noise associated with SPH, but it is also clear (from Fig. 3) that it also changes the shock structure in the disc. Maybe it is not that surprising that the results differ slightly. These results are, however, not consistent with the suggestion (Meru & Bate 2012) that – even for large particle numbers – artificial dissipation provides ~ 30 per cent of the heating when $\beta_{\text{SPH}} = 0.2$.

4 FARGO

In addition to considering how self-gravitating discs evolve in SPH simulations, Meru & Bate (2012) have extended this to consider grid-based simulations of self-gravitation accretion discs. They use the FARGO code and also show that the fragmentation boundary does not converge as resolution increases. In this work, they also vary the artificial viscosity parameter so as to maximize the cooling time at which fragmentation occurs. There are, however, some issues with how they have implemented these FARGO simulations.

As with SPH, grid-based methods such as FARGO (Masset 2000) require a form of artificial viscosity to handle shocks. This is a direct consequence of Godunov’s theorem (Godunov 1954): any numerical scheme that is better than first-order accurate will introduce unphysical oscillations in the flow near shocks. Since almost all numerical methods for gas dynamics aim for at least second-order accuracy, a special recipe is needed around discontinuities in the flow. Finite-difference methods like ZEUS (Stone & Norman 1992) and FARGO employ a Von Neumann–Richtmeyer type of artificial

viscous pressure, which, when considering an axisymmetric disc, takes the form

$$P_{\text{av}} = \begin{cases} Q_{\text{av}}^2 \Sigma \left(\frac{\partial v_r}{\partial r} \right)^2 & \text{if } \partial v_r / \partial r < 0, \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

where Σ denotes the surface density, v_r is the radial velocity and Q_{av} is the artificial viscosity parameter, which has dimensions of length. The reduced artificial viscosity parameter $q = Q_{\text{av}} / \Delta r$, where Δr is the grid spacing, determines over how many grid points shocks will be smeared out. In this view, of course, only values of q larger than unity make sense, and the standard value in FARGO is $q = 1.41$. Choosing a non-linear viscous pressure as artificial viscosity results in the correct entropy jump across shocks and the correct shock propagation velocity (Von Neumann & Richtmeyer 1950).

The form of artificial viscosity given in equation (13) has two important properties: it acts only when the flow is compressed,¹ and it acts, for q of order unity, only on length-scales of the order of the grid scale. The latter property implies that if the value of q makes a difference in the outcome of a simulation, the flow *must* be under-resolved.

We illustrate the effect of artificial viscosity on two one-dimensional (axisymmetric) problems below, one linear and one non-linear.

4.1 Linear problem

For the first problem, we take an equilibrium inviscid Keplerian disc, extending from $r = 0.5$ to 1.5 , with $H/r = 0.05$ at $r = 1$, and add a radial velocity perturbation

$$v_r = 0.01 \exp \left(-\frac{(r-1)^2}{0.001} \right). \quad (14)$$

Note that this is a velocity perturbation equal to 20 per cent of the sound speed. Therefore, no shocks form in this problem, which means that entropy should be materially conserved. For simplicity, we take the surface density to be constant initially, and choose the pressure so that the initial state has constant entropy ($P \propto \Sigma^\gamma$, where we take the ratio of specific heats $\gamma = 1.4$), which means that in an ideal world, the quantity $s = P / \Sigma^\gamma$ should remain constant. No numerical method is ideal, of course, and there are two sources of changes in s : one is due to the finite size of the grid cells, which, unless a special entropy-conserving integration scheme is adopted, will lead to spurious changes in s , and the other is artificial viscosity, which directly changes the entropy through the viscous heating term.

The results after integrating to $t = 2.2$ are displayed in Fig. 5 for three different grid sizes N , with corresponding resolutions $\Delta r = 1/N$. The initial Gaussian pulse in velocity is resolved by ~ 3 cells for $N = 100$, ~ 300 cells for $N = 1000$ and ~ 3000 cells for $N = 10000$. As the artificial viscosity is increased, the maximum change in s increases due to viscous heating, as expected.

The increase in entropy at $q = 0$ is due to the grid only. At the lowest resolution, the effects of the grid and the artificial viscosity are of similar magnitude. FARGO, like most grid-based methods, is second-order accurate. This means that after a fixed number of time steps, errors should decrease as Δr^2 as the grid is refined. Since the

¹ Therefore, unlike as suggested in Meru & Bate (2012), artificial viscosity does *not* act on the Keplerian shear. This is still true if a tensor form of the artificial viscosity is used, as long as the off-diagonal terms of the stress tensor are dropped (Stone & Norman 1992).

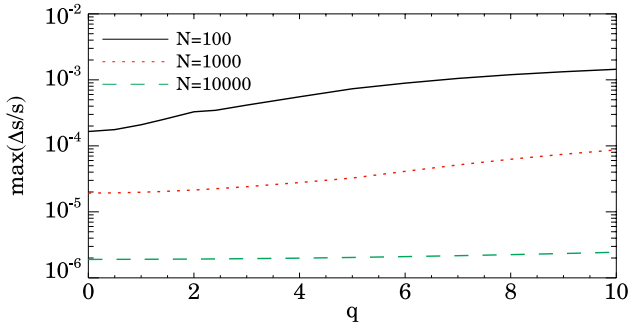


Figure 5. Maximum fractional change in entropy in the linear test problem as a function of the artificial viscosity parameter q , for three different grid sizes.

number of time steps required to reach $t = 2.2$ is proportional to N , we expect the errors due to the grid at $t = 2.2$ to decrease as Δr , which is exactly what is observed in Fig. 5.

As the resolution is increased, the differences between runs with $q = 0$ and $q \sim 1$ decrease. For all resolutions except $N = 100$, taking $q \sim 1$ makes no difference compared to $q = 0$. In other words, for $N = 1000$ and 10000 , heating due to artificial viscosity is completely negligible for $q \sim 1$. Only for $N = 100$ does artificial viscosity make a difference, but this is to be expected, since the extent of the initial pulse is comparable to the grid scale, which means it will feel the artificial viscosity. Therefore, as expected, artificial viscosity plays no role in heating a smooth flow, unless it is not resolved.

4.2 Non-linear problem

As a second test problem, we set up a non-linear wave characterized by initial conditions

$$\Sigma = \begin{cases} 1 & \text{if } r < 1, \\ 0.1 & \text{otherwise,} \end{cases} \quad (15)$$

in the same inviscid equilibrium Keplerian disc as above. The initial pressure is set again such that the initial state is isentropic. The solution develops a shock that leads to an increase in entropy. The correct increase in s was measured from a simulation using a Riemann solver (Paardekooper & Mellema 2006) at $N = 10000$.

The results obtained with FARGO are displayed in Fig. 6. It is immediately clear that simulations with $q \leq 1$ strongly underestimate the change in entropy. This is again not surprising, since artificial viscosity is *necessary* in this case because of the presence of an

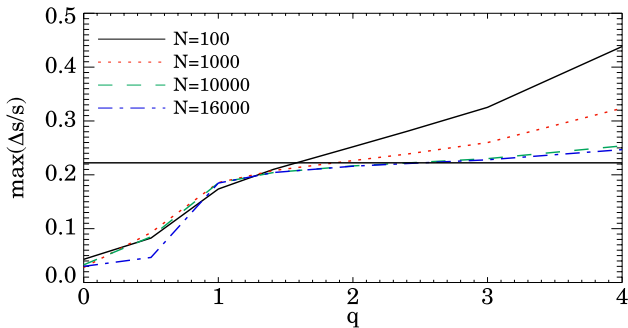


Figure 6. Maximum fractional change in entropy for the non-linear test problem as a function of the artificial viscosity parameter q , for four different grid sizes. The horizontal line shows the solution obtained with a Riemann solver at $N = 10000$.

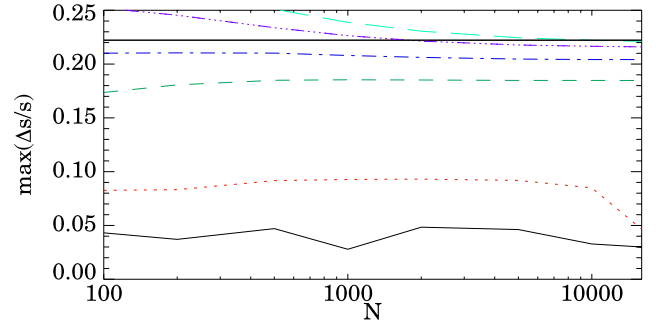


Figure 7. Maximum fractional change in entropy for the non-linear test problem as a function of the grid size N . From bottom to top, $q = 0, 0.5, 1.0, 1.41, 2.0, 2.41$. The horizontal line shows the solution obtained with a Riemann solver at $N = 10000$.

entropy-generating shock. Similar to SPH, choosing the artificial viscosity parameter too high leads to unphysical, artificial heating: choosing $q > 2$ leads to too much viscous heating for $N = 100$, but at high enough resolution a plateau emerges giving roughly the same amount of entropy generation independent of q .

In Fig. 7, we look at the same problem but now as a function of resolution. Simulations with $q < 2$ systematically underestimate the entropy production, independent of resolution. The situation is most severe for $q = 0$ and 0.5 , which do not even show convincing signs of convergence with resolution. Simulations with $q \geq 1$ do seem to converge, but to a level that depends on q . For the largest values of q , the entropy increase converges to a value very close to the correct one, but at the price of overestimating the entropy increase at low resolution. The standard value $q = 1.41$ seems to be a good compromise.

4.3 Implications for self-gravitating disc simulations

It is not completely straightforward to translate the above results to two dimensions. In this case, an extra source of error comes from dimensional splitting, the effects of which are not entirely clear especially if the FARGO algorithm is used (Masset 2000). Moreover, shocks are no longer necessarily aligned with the grid, which in all likelihood changes the dissipation properties of the grid. However, a few general statements can be made.

The ‘optimum’ value of q is larger than unity. For example, $q = 1.41$ gives artificial viscosity that leads to a good estimate of entropy production in shocks (Fig. 6), while its effect on *resolved* smooth flow is negligible (Fig. 5). Values of $q < 1$ do not give the correct amount of shock heating, and therefore cannot be expected to give physical results. Note that the ‘optimum’ value chosen by Meru & Bate (2012), $q = 0.5$, underestimates the entropy increase in a shock by a factor of 3 (see Fig. 7). This will have serious consequences for the simulation results if heating is due to shock dissipation, which is the case for self-gravitating discs.

If the amount of artificial viscosity makes a difference in the results, the flow is under-resolved. This can be due to shocks, in which case there the flow in fact cannot be resolved, or due to unresolved smooth flow. In the latter case, reducing the amount of artificial viscosity will in general not lead to a much better solution, since errors due to the finite size of the grid cells are likely to be as large as the error introduced by artificial viscosity, precisely because the flow is unresolved (Fig. 5). Moreover, reducing the amount of artificial viscosity can only be safely done when it is absolutely certain that no shocks are present in the problem, which, for self-gravitating discs, we know is not the case.

Keeping the artificial parameter fixed at $q = 1.41$, there are still several avenues for investigating the problem of convergence in grid-based simulations. A direct comparison between 2D global and local simulations (Gammie 2001), which do not appear to show convergence (Paardekooper 2012), is definitely warranted. Care must be taken in global simulations to avoid initial transients (Paardekooper et al. 2011). It may be that the lack of convergence in grid-based simulations is the result of the two-dimensional approximation, even though smoothing of the gravitational potential does not seem to make much of a difference (Paardekooper 2012). It may also be that the inviscid problem is ill-posed, and that a finite amount of (Navier–Stokes) viscosity is needed to reach convergence.

5 IMPORTANCE OF ARTIFICIAL VISCOSITY IN SPH AND FARGO

Artificial viscosity is a necessary feature in numerical simulations in order to make the code behave in a reasonable way at the smallest resolvable scale, which is the scale of the grid in grid-based codes and the smoothing length in SPH. Without artificial viscosity, particle interpenetration would make the outcome of any SPH simulation useless. The use of a grid introduces its own associated ‘viscosity’, which acts on the smallest resolvable scale (the grid scale) and may or may not behave like a real viscosity. This is why simulations of turbulence often employ an additional physical Navier–Stokes viscosity to make sure that energy dissipation on the smallest scales is well behaved and physical (see e.g. Fromang et al. 2007). In addition, both SPH and grid-based codes like *FARGO* need artificial viscosity to handle shocks correctly.

It should be clear from the discussion in Sections 3 and 4 that care should be taken when trying to adjust the amount of artificial viscosity for a particular problem, both in grid-based codes and SPH. When the level of artificial viscosity makes a difference in the simulation outcome, this means that features close to the smallest resolvable scale (either the grid scale or the smoothing length) play an important role. Artificial viscosity is a way of converting bulk motion on these small scales into heat. This can be unphysical, if the underlying flow is smooth, or physical, in the case of shocks. Of course, one would always like to reduce the amount of artificial viscosity as much as possible. However, reducing the amount of artificial viscosity below the level required to handle shocks correctly, e.g. taking $q < 1$ in *FARGO*, can only be done if it is known that no shocks will occur in the problem. And even then, the results in Section 4 indicate that for smooth flow dissipation is dominated by the grid rather than artificial viscosity, so that changing q will not make a difference in the simulation outcome.

The only way to do a better job for smooth flow in a grid-based simulation is to increase the resolution. In SPH, there are more artificial viscosity parameters to play with, and it may be slightly less clear to what extent the choices for α_{SPH} and β_{SPH} are free. Since there is no dissipation due to a grid, it is likely that the level of artificial viscosity matters more compared to grid-based codes. Typically SPH simulations use $\beta_{\text{SPH}} = 2\alpha_{\text{SPH}}$. This is so that the linear term dominates when particles are converging slowly, and the quadratic term dominates when the particles are converging fast enough that shocks are likely to form. Using $\beta_{\text{SPH}} = 20\alpha_{\text{SPH}}$, as suggested by Meru & Bate (2012), effectively means that the quadratic term is likely to always dominate and, hence, is likely to change the properties of the simulation itself.

In the case where shocks are present, the situation is more complicated, since the numerical method will always smear shocks over a few grid cells, for a grid-based code, or a few smoothing lengths

for SPH. Increasing the resolution therefore keeps reducing the size of shocks, which means that ever smaller scales are present in the problem. These smallest scales can interact in non-trivial ways with larger scale structures (clumps, waves), and it may not be immediately clear whether convergence can be reached and at what resolution.

6 DISCUSSION AND CONCLUSIONS

Recently, Meru & Bate (2011) have suggested that, in three-dimensional SPH simulations of self-gravitating accretion disc with β -cooling, the cooling time (β_{cool}) at which fragmentation occurs does not converge as resolution is increased. They have extended this work (Meru & Bate 2012) to suggest that with typical artificial viscosity parameters, the fragmentation boundary is converging towards a critical cooling time of $\beta_{\text{crit}} = 17.4$. They go on to argue, however, that adjusting the artificial viscosity parameters (so as to maximize the critical cooling time, β_{crit}) suggests that, with an appropriate choice of the artificial viscosity parameters, the simulations actually converge towards $\beta_{\text{crit}} = 29.2$.

Meru & Bate (2012) then continue this by considering the evolution of self-gravitating disc using the grid-based code *FARGO*. Here they also vary the artificial viscosity parameter so as to maximize the critical cooling time and show that these simulations also do not converge as resolution increases.

It has been suggested (Rice et al. 2012) that the non-convergence seen in Meru & Bate (2011) was simply a consequence of the manner in which cooling was implemented. We have extended the work of Rice et al. (2012) here to show that by implementing what they call smoothed cooling, fragmentation requires $\beta_{\text{cool}} < 8$ for all resolutions considered (from 250 000 particles to 10 million particles). This is more consistent with other work (Gammie 2001) and also makes physical sense given that the Jeans mass is well resolved in most of the disc for simulations with 500 000 particles or more, and that artificial viscosity should be providing less than 10 per cent of the heating in such simulations, so should not be significantly influencing the fragmentation boundary. Furthermore, if fragmentation can occur for $\beta_{\text{cool}} \sim 30$, this suggests that a clump can contract and become bound even though the time-scale over which it is losing energy is significantly greater than the orbital period, which is likely to determine the time-scale over which we would expect the clump to heat.

We also consider how the alternative artificial viscosity values suggested by Meru & Bate (2012) influence the results when using smoothed cooling. We find that the results are consistent with those obtained using the original artificial viscosity parameters. Rather than increasing the critical cooling time, β_{crit} , by 50 per cent (Meru & Bate 2012), fragmentation still requires $\beta_{\text{cool}} < 8$, for all particle numbers considered. If this change to the artificial viscosity parameters was reducing the level of artificial heating (as suggested by Meru & Bate 2012), then we would expect the results to be independent of the implementation of the cooling. That they are not suggests that changing these parameters is influencing the simulations in some numerical way, rather than simply changing the level of artificial heating – especially as the expectation is that the changes made by Meru & Bate (2012) should have increased, rather than reduced, the level of artificial heating. This is also consistent with the significant difference between the spiral shock structure in the disc with $\beta_{\text{SPH}} = 2$ when compared to simulations with $\beta_{\text{SPH}} = 0.2$.

What was quite attractive about the Meru & Bate (2012) work was that they obtained very similar results when using the grid-based *FARGO* code. However, here, they also varied the artificial

viscosity parameter so as to minimize the level artificial viscosity. As discussed earlier, however, the artificial viscosity in FARGO only acts on converging flows and so should not produce any artificial (non-shock related) heating. Additionally, in order to produce good estimates of entropy production at shocks, it requires that, as discussed earlier, the optimal value for the artificial viscosity in FARGO be larger than unity. Meru & Bate (2012) claim that the optimal value is $q = 0.5$, much smaller than would be regarded as suitable for such a simulation. Admittedly, even their simulations with $q = 1.41$ did not show signs of convergence, but this could be related to unsuitable initial conditions (Paardekooper et al. 2011) or the possibility of stochasticity (Paardekooper 2012) and should be investigated further.

Essentially, the SPH simulations presented here show that if one implements the cooling so as to remove the unphysical discontinuity at the contact discontinuity behind shocks (Price 2008a,b), we appear to get convergence as the resolution increases, and the fragmentation boundary that we determine is consistent with earlier work (fragmentation occurring for cooling times between $\beta_{\text{cool}} = 6$ and 8 for $\gamma = 5/3$). Although, we have not investigated Meru & Bate (2012)'s FARGO results in as much detail, it seems clear that what they regard as the optimal value for the artificial viscosity parameter ($q = 0.5$) is well below what would be regarded as acceptable for such simulations. This might suggest that their FARGO results suffer from additional numerical issues. With the exception of the possibility of stochastic fragmentation (Paardekooper 2012), we therefore conclude that there is no real evidence that fragmentation can occur in self-gravitating discs with long cooling times and that the likely fragmentation boundary is similar to that suggested by earlier work (Gammie 2001; Rice et al. 2005). Consequently, this implies that – as suggested by earlier studies (Rafikov 2005; Stamatellos & Whitworth 2008; Clarke 2009; Rice & Armitage 2009) – gas giant planet formation via disc fragmentation is unlikely in the inner regions ($r < 50$ au) of protostellar discs.

ACKNOWLEDGEMENTS

All simulations presented in this work were carried out using high-performance computing funded by the Scottish Universities Physics Alliance (SUPA). WKMR and DHF gratefully acknowledge support from STFC grant ST/J001422/1. PJA acknowledges support from NASA's Astrophysics Theory and Origins of Solar Systems programs through grants NNX11AE12G and NNX13AI58G.

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